CHAPTER 4 Matrices

EXERCISE 4a (p. 54)

1. 2 x 2
2. 2 x 3
3. 2 x 1
4. 1 x 1
5. 1 x 3
6. 3 x 2

7. a) 6 b) 8 c) 2 d) 7

8. $\begin{matrix}3\\1\\7\end{matrix}\ ;\ \begin{matrix}4\\7\\2\end{matrix}$ a) 7 b) 6 c) 4

9. $\begin{pmatrix}0&0&0\\1&1&1\\2&2&2\end{pmatrix}$

10. $\begin{pmatrix}3&1\\3&1\\3&1\end{pmatrix}$

EXERCISE 4b (p. 56)

1. $\begin{pmatrix}12\\15\end{pmatrix}$

2. $\begin{pmatrix}15&4\\7&1\end{pmatrix}$

3. Not possible

4. (9, 5)

5. $\begin{pmatrix}11&2&2\\6&7&7\end{pmatrix}$

6. $\begin{pmatrix}11&11\\11&5\end{pmatrix}$

7. (5 3 5)

8. Not possible

9. $\begin{pmatrix}6&8\\7&7\\7&7\end{pmatrix}$

10. (10 8)

11. $\begin{pmatrix}1&8\\-4&7\end{pmatrix}$

12. $\begin{pmatrix}-2\\2\end{pmatrix}$

13. $\begin{pmatrix}-2&7\\-5&3\end{pmatrix}$

14. (4 6)

15. $\begin{pmatrix}-3\\-3\\6\end{pmatrix}$

16. Not possible

17. $\begin{pmatrix}2&10\\5&-3\end{pmatrix}$

18. $\begin{pmatrix}5&-5\\3&0\end{pmatrix}$

19. $\begin{pmatrix}0&8\\8&-2\end{pmatrix}$

20. Not possible

21. Not possible

22. (1 6 –3)

23. $\begin{pmatrix}2&3&4\\5&0&-12\end{pmatrix}$

EXERCISE 4c (p. 58)

1. $\begin{pmatrix}3\\6\\12\end{pmatrix}$

2. $\begin{pmatrix}2&8&0\\4&-2&6\end{pmatrix}$

3. $\begin{pmatrix}1&2\\\frac{1}{2}&3\\1\frac{1}{2}&4\end{pmatrix}$

4. $\begin{pmatrix}6&24\\18&-12\end{pmatrix}$

5. $\begin{pmatrix}-6&-30\\6&12\end{pmatrix}$

6. $\begin{pmatrix}4&0\\\frac{2}{3}&1\frac{1}{3}\\2&3\frac{1}{3}\end{pmatrix}$

7. $\begin{pmatrix} 2 & -2 \\ 1 & 3 \end{pmatrix}$

8. Not possible

9. $\begin{pmatrix} -3 \\ 3 \\ 0 \end{pmatrix}$

10. $\begin{pmatrix} 2 & 4 & 2 \\ -3 & -3 & -1 \end{pmatrix}$

11. Not possible

12. $\begin{pmatrix} -3 & -1 & 2 \\ 9 & 5 & 4 \\ 1 & 11 & 5 \end{pmatrix}$

EXERCISE 4d (p. 59)

1. $\begin{pmatrix} -1 & 8 \\ 6 & 1 \end{pmatrix}$

2. $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

3. Not possible

4. $\begin{pmatrix} 7 & -1 \\ 5 & -1 \end{pmatrix}$

5. $\begin{pmatrix} 1\frac{1}{3} \\ 1\frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$

6. $\begin{pmatrix} -3 \\ -3 \\ -3 \end{pmatrix}$

7. $\begin{pmatrix} 2 & 8 \\ 6 & 4 \end{pmatrix}$

8. $\begin{pmatrix} -1 & 2 \\ 1\frac{1}{2} & -\frac{1}{2} \end{pmatrix}$

9. $\begin{pmatrix} 8 \\ 9 \\ 3 \end{pmatrix}$

10. $\begin{pmatrix} 24 & 8 & -4 \\ 16 & 12 & 16 \end{pmatrix}$

11. Not possible

12. $\begin{pmatrix} 8 & -2 & -2 \\ 1 & 4 & 4 \end{pmatrix}$

EXERCISE 4e (p. 61)

A vector can be represented by a column matrix. Capital letters are used to denote matrices, including 2 x 1 column matrices, e.g. $\mathbf{A} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 5 & 2 \\ -4 & 3 \end{pmatrix}$, but a lower case letter is used when a column matrix represents a vector, e.g. $\mathbf{a} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$.

1. **B** 2 x 1, **C** 2 x 2, **D** 2 x 2, **E** 1 x 3, **F** 1 x 2, **G** 2 x 3

2. $\begin{pmatrix} 9 & 4 & 4 \\ 7 & 1 & 7 \end{pmatrix}$

3. Not possible

4. $\begin{pmatrix} 4 & -1 \\ 0 & 6 \end{pmatrix}$

5. $\begin{pmatrix} 12 & 9 & 3 \\ 3 & 6 & 9 \end{pmatrix}$

6. Not possible

7. $(1\frac{1}{2} \quad 1)$

8. Not possible

9. $\begin{pmatrix} 1 & -2 & 2 \\ 5 & -3 & 1 \end{pmatrix}$

10. $\begin{pmatrix} 24 \\ 6 \end{pmatrix}$

11. $\begin{pmatrix} 4\frac{1}{2} & 1\frac{1}{2} \\ \frac{3}{4} & 3 \end{pmatrix}$

12. Not possible

13. Not possible

EXERCISE 4f (p. 62)

Here are two methods for remembering the order of matrix multiplication: (1) Calling the process "row-column" multiplication helps emphasize that rows are taken from the first matrix and columns from the second. (2) The picture of a person running along a diving board and then diving downwards gives the idea of "row first and then column".

1. $\begin{pmatrix} 29 \\ 27 \end{pmatrix}$ **3.** $\begin{pmatrix} 5 \\ 7 \end{pmatrix}$ **5.** $\begin{pmatrix} 9 \\ 5 \end{pmatrix}$ **7.** $\begin{pmatrix} 26 \\ 10 \end{pmatrix}$ **9.** $\begin{pmatrix} 56 \\ 49 \end{pmatrix}$

2. $\begin{pmatrix} 14 \\ 11 \end{pmatrix}$ **4.** $\begin{pmatrix} 9 \\ 2 \end{pmatrix}$ **6.** $\begin{pmatrix} 18 \\ 14 \end{pmatrix}$ **8.** $\begin{pmatrix} 58 \\ 19 \end{pmatrix}$ **10.** $\begin{pmatrix} 26 \\ 10 \end{pmatrix}$

EXERCISE 4g (p. 64)

1. $\begin{pmatrix} 7 & \\ 10 & \end{pmatrix}$

2. $\begin{pmatrix} 14 & \\ & 22 \end{pmatrix}$

3. $\begin{pmatrix} & 37 \\ & 2 \end{pmatrix}$

4. $\begin{pmatrix} 23 & 16 \\ & \end{pmatrix}$

5. $\begin{pmatrix} 19 & \\ 22 & \end{pmatrix}$

6. $\begin{pmatrix} & 16 \\ 12 & \end{pmatrix}$

7. $\begin{pmatrix} & 12 \\ & 3 \end{pmatrix}$

8. $\begin{pmatrix} 17 & 19 \\ 5 & \end{pmatrix}$

9. $\begin{pmatrix} 22 & 52 \\ 10 & 22 \end{pmatrix}$

10. $\begin{pmatrix} 44 & 32 \\ 8 & 7 \end{pmatrix}$

11. $\begin{pmatrix} 16 & 14 \\ 11 & 14 \end{pmatrix}$

12. $\begin{pmatrix} 8 & 18 \\ 18 & 36 \end{pmatrix}$

13. $\begin{pmatrix} 44 & 40 \\ 18 & 31 \end{pmatrix}$

14. $\begin{pmatrix} 21 & 7 \\ 17 & 9 \end{pmatrix}$

15. $\begin{pmatrix} 0 & 14 \\ 10 & 8 \end{pmatrix}$

16. $\begin{pmatrix} 15 & 20 \\ 5 & 0 \end{pmatrix}$

17. $\begin{pmatrix} 3 & -4 \\ 13 & 6 \end{pmatrix}$

18. $\begin{pmatrix} -24 & -17 \\ -10 & -9 \end{pmatrix}$

19. $\begin{pmatrix} 21 & 11 \\ 9 & 2 \end{pmatrix}$

20. $\begin{pmatrix} -16 & 1 \\ -6 & -1 \end{pmatrix}$

EXERCISE 4h (p. 66)

1. $\begin{pmatrix} 20 & 13 \\ 8 & 5 \end{pmatrix}$

2. $\begin{pmatrix} 10 & 7 \\ 22 & 15 \end{pmatrix}$

3. $\begin{pmatrix} 31 & 35 \\ 15 & 17 \end{pmatrix}$

4. $\begin{pmatrix} 44 & 29 \\ 6 & 4 \end{pmatrix}$

5. $\begin{pmatrix} 8 & 6 \\ 4 & 2 \end{pmatrix}$

6. $\begin{pmatrix} 8 & 6 \\ 4 & 2 \end{pmatrix}$

7. $\begin{pmatrix} 15 & 17 \\ 31 & 35 \end{pmatrix}$

8. $\begin{pmatrix} 46 & 31 \\ 6 & 4 \end{pmatrix}$

9. $\begin{pmatrix} 4 & 2 \\ 8 & 6 \end{pmatrix}$

10. $\begin{pmatrix} 4 & 2 \\ 8 & 6 \end{pmatrix}$

11. $\begin{pmatrix} 14 & 16 \\ 2 & 2 \end{pmatrix}$

12. $\begin{pmatrix} 14 & 16 \\ 2 & 2 \end{pmatrix}$

13. One of the two matrices was **D**

EXERCISE 4i (p.68)

1. $\begin{pmatrix} 7 \\ 10 \end{pmatrix}$

2. $\begin{pmatrix} 13 \\ 32 \end{pmatrix}$

3. (10)

4. $\begin{pmatrix} 20 & 10 \\ 70 & 23 \end{pmatrix}$

5. $\begin{pmatrix} 32 & 26 & 16 \\ 20 & 19 & 11 \end{pmatrix}$

6. $\begin{pmatrix} 24 \\ 33 \\ 42 \end{pmatrix}$

7. $\begin{pmatrix} 21 & 39 & 8 \\ 17 & 26 & 7 \end{pmatrix}$

8. $\begin{pmatrix} 10 & 11 \\ 36 & 30 \\ 31 & 28 \end{pmatrix}$

9. $(13 \quad 31 \quad 27)$

10. (15)

EXERCISE 4j (p.70)

1. $2 \times \boxed{2 \quad 2} \times 1 = 2 \times 1;$ $\begin{pmatrix} 7 \\ 6 \end{pmatrix}$

2. $2 \times \boxed{3 \quad 3} \times 1 = 2 \times 1;$ $\begin{pmatrix} 22 \\ 12 \end{pmatrix}$

3. $1 \times \boxed{2 \quad 2} \times 1 = 1 \times 1;$ (10)

4. $2 \times \boxed{3 \quad 3} \times 2 = 2 \times 2;$ $\begin{pmatrix} 20 & 10 \\ 70 & 23 \end{pmatrix}$

5. $2 \times \boxed{2 \quad 2} \times 2 = 2 \times 2;$ $\begin{pmatrix} 11 & 20 \\ 24 & 43 \end{pmatrix}$

6. $2 \times \boxed{1 \quad 1} \times 2 = 2 \times 2;$ $\begin{pmatrix} 3 & 4 \\ 6 & 8 \end{pmatrix}$

7. $1 \times \boxed{2 \quad 2} \times 2 = 1 \times 2;$ $(21 \quad 36)$

8. $3 \times \boxed{1 \quad 1} \times 3 = 3 \times 3;$ $\begin{pmatrix} 4 & 5 & 6 \\ 8 & 10 & 12 \\ 12 & 15 & 18 \end{pmatrix}$

9. $\begin{pmatrix} 16 \\ 6 \end{pmatrix}$

10. Not possible

11. $\begin{pmatrix} 11 & 20 \\ 24 & 43 \end{pmatrix}$

12. Not possible

13. $\begin{pmatrix} 15 & 4 & 3 \\ 48 & 13 & 12 \end{pmatrix}$

14. Not possible

15. (30)

16. Not possible

17. $(3 \quad 24)$

18. $\begin{pmatrix} 6 & 12 & 15 \\ 8 & 16 & 20 \\ 2 & 4 & 5 \end{pmatrix}$

EXERCISE 4k (p. 71)

1. $\begin{pmatrix} 6 \\ 5 \end{pmatrix}$

2. $\begin{pmatrix} 10 \\ -19 \end{pmatrix}$

3. $(-2 \quad -6)$

4. $\begin{pmatrix} 1 \\ 1 \\ -22 \end{pmatrix}$

5. $\begin{pmatrix} 8 & -26 \\ -16 & -17 \end{pmatrix}$

6. $(-38 \quad 12)$

7. (-26)

8. $\begin{pmatrix} -24 & -4 & 12 \\ 6 & 1 & -3 \\ 6 & 1 & -3 \end{pmatrix}$

9. $\begin{pmatrix} 7 & 18 & -1 \\ -7 & -18 & 1 \end{pmatrix}$

10. $\begin{pmatrix} 12 & 18 \\ 8 & 12 \\ -6 & -9 \end{pmatrix}$

EXERCISE 4l (p. 72)

1. $\begin{pmatrix} 13 & -8 \\ 7 & -2 \end{pmatrix}$

2. $\begin{pmatrix} 2 & -1 \\ 12 & 9 \end{pmatrix}$

3. $\begin{pmatrix} 10 \\ 5 \end{pmatrix}$

4. Not possible

5. $\begin{pmatrix} 3 & 4 \\ 6 & 8 \end{pmatrix}$

6. Not possible

7. Not possible

8. Not possible

9. Not possible

10. (11)

11. $(14 \quad 6)$

12. Not possible

13. Not possible

14. $(9 \quad 12)$

15. Not possible

16. **AA** = $\begin{pmatrix} 19 & 18 \\ 6 & 7 \end{pmatrix}$

AC = $\begin{pmatrix} 21 & 8 \\ 4 & 2 \end{pmatrix}$

BB = $\begin{pmatrix} -5 & -2 \\ 3 & -6 \end{pmatrix}$

BC = $\begin{pmatrix} 8 & 2 \\ 18 & 6 \end{pmatrix}$

BD = $\begin{pmatrix} -3 \\ 3 \end{pmatrix}$

CA = $\begin{pmatrix} 26 & 22 \\ -4 & -3 \end{pmatrix}$

CB = $\begin{pmatrix} 12 & -12 \\ -1 & 2 \end{pmatrix}$

CC = $\begin{pmatrix} 34 & 12 \\ -6 & -2 \end{pmatrix}$

CD = $\begin{pmatrix} 10 \\ -1 \end{pmatrix}$

DH = $\begin{pmatrix} 3 \\ 6 \end{pmatrix}$

EA = $(16 \quad 17)$

EB = $(15 \quad -6)$

FE = $\begin{pmatrix} 18 & 24 \\ 3 & 4 \\ -12 & -16 \end{pmatrix}$

FH = $\begin{pmatrix} 18 \\ 3 \\ -12 \end{pmatrix}$

GF = $\begin{pmatrix} -4 \\ 15 \\ 5 \end{pmatrix}$

GG= $\begin{pmatrix} 18 & -2 & 17 \\ -13 & 35 & 11 \\ 23 & 4 & 27 \end{pmatrix}$

HH = (9)

EXERCISE 4m (p. 73)

1. $\begin{pmatrix} 8 & 2 \\ -21 & -8 \end{pmatrix}$

2. $\begin{pmatrix} 7 & 4 \\ -3 & 3 \end{pmatrix}$

3. $\begin{pmatrix} -5 & 0 \\ -5 & 3 \end{pmatrix}$

4. $\begin{pmatrix} 5 & 0 \\ 5 & -3 \end{pmatrix}$

5. Not possible

6. Not possible

7. (–2 –3)

8. $\begin{pmatrix} -2 & 1 \\ 8 & 12 \end{pmatrix}$

9. $\begin{pmatrix} 5 & 3 & 3 \\ 11 & 5 & 9 \\ 2 & 2 & 0 \end{pmatrix}$

10. Not possible

11. Not possible

12. (4 2)

EXERCISE 4n (p. 74)

1. $\begin{pmatrix} 12 & 10 \\ -2 & 13 \end{pmatrix}$

2. $\begin{pmatrix} 78 & -10 \\ 31 & -13 \end{pmatrix}$

3. $\begin{pmatrix} 13 & -6 \\ 18 & 1 \end{pmatrix}$

4. $\begin{pmatrix} 68 & 16 \\ 61 & 4 \end{pmatrix}$

5. $\begin{pmatrix} 48 & 40 \\ 38 & 17 \end{pmatrix}$

6. $\begin{pmatrix} 34 & -25 \\ 75 & -16 \end{pmatrix}$

7. $\begin{pmatrix} 78 & 8 \\ 31 & 28 \end{pmatrix}$

8. $\begin{pmatrix} 50 & 27 \\ 32 & 56 \end{pmatrix}$

9. $\begin{pmatrix} 55 & -11 \\ 66 & -22 \end{pmatrix}$

10. $\begin{pmatrix} -16 & 56 \\ -56 & 20 \end{pmatrix}$

11. $\begin{pmatrix} -64 & 0 \\ 0 & -64 \end{pmatrix}$

12. $\begin{pmatrix} 68 & -12 \\ 61 & -7 \end{pmatrix}$

EXERCISE 4p (p.74)

1. 2 x 2 and 2 x 1

2. Yes

3. **A, C** are compatible but not **C, A**

4. $\begin{pmatrix} 23 & -11 \\ 19 & -13 \end{pmatrix}$

5. $\mathbf{A}^2 = \begin{pmatrix} 27 & 18 \\ 9 & 18 \end{pmatrix}$. It is not possible to find $\mathbf{C}^2$

6. Not possible

7. $\begin{pmatrix} 9 & -3 \\ 12 & -9 \end{pmatrix}$

8. $\begin{pmatrix} 13 & 3 \\ 6 & 5 \end{pmatrix}$

9. 4

10. **BC**

EXERCISE 4q (p. 75)

1. $\begin{pmatrix} 4 & 2 & -2 \\ 8 & 6 & 2 \end{pmatrix}$

2. Not possible

3. Not possible

4. 2 x 3 and 2 x 2

5. No

6. 3

7. 1

8. **QP**

9. $\begin{pmatrix} 17 \\ 13 \end{pmatrix}$

10. It is not possible to find $\mathbf{P}^2$. $\mathbf{Q}^2 = \begin{pmatrix} 7 & 14 \\ -7 & 14 \end{pmatrix}$