

CHAPTER 13 Quadratic Equations

This chapter introduces quadratic equations and covers solution by factorisation. Graphical solution, completing the square and using the formula are in Book 4A.

Many of the questions in the first two exercises can be considered orally. They form a useful introduction. It is also worth pointing out that if $A \times B = 0$ then stating that either $A = 0$ or $B = 0$ does not rule out the possibility that both A and B are zero.

Exercise 13a (p. 222)

- | | | | | | |
|----------|------|------|-----------|------|-------|
| 1. a) 8 | b) 0 | c) 0 | 6. a) 33 | b) 0 | c) 0 |
| 2. a) 0 | b) 5 | c) 0 | 7. a) -24 | b) 0 | c) 0 |
| 3. a) 0 | b) 7 | c) 0 | 8. a) 70 | b) 0 | c) 0 |
| 4. a) 0 | b) 0 | c) 3 | 9. a) 0 | b) 0 | c) 20 |
| 5. a) 20 | b) 0 | c) 6 | 10. a) -9 | b) 0 | c) 0 |

Exercise 13b (p. 223)

- | | | |
|--------------|------------------------|-------------------------|
| 1. 0 | 10. 7 | 19. $a = 0$ or $b = 2$ |
| 2. 0 | 11. any value | 20. $a = 3$ or $b = 0$ |
| 3. 0 | 12. 0 | 21. $a = 9$ or $b = 0$ |
| 4. any value | 13. a) 0 b) 0 | 22. $a = 0$ or $b = 4$ |
| 5. 4 | 14. a) 0 b) 0 | 23. $a = 0$ or $b = 10$ |
| 6. 1 | 15. a) 0 b) 0 | 24. $a = 1$ or $b = 0$ |
| 7. 0 | 16. a) 0 b) any value | 25. $a = 7$ or $b = 0$ |
| 8. 2 | 17. $a = 0$ or $b = 1$ | 26. $a = 12$ or $b = 0$ |
| 9. 0 | 18. $a = 0$ or $b = 5$ | |

Exercise 13c (p. 225)

- | | | |
|-------------|--------------|------------------|
| 1. 0 or 3 | 11. 1, 2 | 21. -1 or -8 |
| 2. 0 or 5 | 12. 5 or 9 | 22. p or q |
| 3. 0 or 3 | 13. 7 or 10 | 23. $-a$ or $-b$ |
| 4. 0 or -4 | 14. 4 or 7 | 24. 4 or -1 |
| 5. 0 or -5 | 15. 1 or 6 | 25. -9 or 8 |
| 6. 0 or 6 | 16. 8 or -11 | 26. -6 or -7 |
| 7. 0 or 10 | 17. 3 or -5 | 27. -10 or -11 |
| 8. 0 or 7 | 18. -7 or 2 | 28. a or b |
| 9. 0 or -7 | 19. -2 or -3 | 29. $-a$ or b |
| 10. 0 or -9 | 20. -4 or -9 | 30. c or $-d$ |

Exercise 13d (p. 226)

- | | | | |
|-----------------------------------|---------------------------------------|-------------------------------------|---------------------------------------|
| 1. 1 or $2\frac{1}{2}$ | 6. $-\frac{2}{5}$ or 7 | 11. $2\frac{1}{3}$ or 2 | 16. $-\frac{3}{4}$ or $2\frac{1}{2}$ |
| 2. 4 or $\frac{2}{3}$ | 7. $-\frac{5}{6}$ or $\frac{2}{3}$ | 12. $1\frac{2}{3}$ or $\frac{1}{2}$ | 17. $-\frac{9}{10}$ or $\frac{4}{5}$ |
| 3. $\frac{4}{5}$ or $\frac{3}{4}$ | 8. $\frac{3}{8}$ or $-2\frac{1}{2}$ | 13. 0 or $\frac{1}{3}$ | 18. $\frac{2}{3}$ or $-2\frac{1}{4}$ |
| 4. 0 or $1\frac{1}{4}$ | 9. $1\frac{1}{7}$ or $-3\frac{3}{4}$ | 14. 0 or $\frac{3}{7}$ | 19. $2\frac{2}{5}$ or $-3\frac{1}{2}$ |
| 5. 0 or $\frac{3}{10}$ | 10. $-\frac{3}{4}$ or $-1\frac{1}{2}$ | 15. $-1\frac{1}{2}$ or 3 | 20. $-1\frac{3}{5}$ or $-\frac{3}{4}$ |

Exercise 13e (p. 226)

- | | | | |
|-------------|--------------|---------------|--------------|
| 1. 1 or 2 | 11. 1 or -7 | 21. -1 or -2 | 31. ± 1 |
| 2. 1 or 7 | 12. 4 or -2 | 22. -1 or -7 | 32. ± 3 |
| 3. 2 or 3 | 13. 3 or -4 | 23. -3 or -5 | 33. ± 4 |
| 4. 2 or 5 | 14. 5 or -3 | 24. -2 or -6 | 34. ± 9 |
| 5. 3 or 4 | 15. 2 or -9 | 25. -2 or -9 | 35. ± 13 |
| 6. 1 or 5 | 16. -1 or 13 | 26. -1 or -6 | 36. ± 2 |
| 7. 1 or 11 | 17. 2 or -3 | 27. -2 or -5 | 37. ± 5 |
| 8. 2 or 4 | 18. -2 or 6 | 28. -1 or -13 | 38. ± 10 |
| 9. 2 or 6 | 19. 4 or -5 | 29. -1 or -15 | 39. ± 12 |
| 10. 1 or 12 | 20. -3 or 8 | 30. -3 or -6 | 40. ± 6 |

Exercise 13f (p. 228)

- | | | | |
|-----------------------|------------------------|-------------------------|--------------------------|
| 1. 0 or 2 | 6. 0 or 5 | 11. 0 or $-\frac{3}{2}$ | 16. 0 or $-\frac{7}{6}$ |
| 2. 0 or 10 | 7. 0 or -3 | 12. 0 or $-\frac{5}{8}$ | 17. 0 or $-\frac{7}{12}$ |
| 3. 0 or -8 | 8. 0 or -1 | 13. 0 or 7 | 18. 0 or -4 |
| 4. 0 or $\frac{1}{2}$ | 9. 0 or $\frac{5}{3}$ | 14. 0 or $-\frac{5}{3}$ | 19. 0 or $\frac{2}{7}$ |
| 5. 0 or $\frac{5}{4}$ | 10. 0 or $\frac{7}{5}$ | 15. 0 or $\frac{12}{7}$ | 20. 0 or $-\frac{3}{14}$ |

Exercise 13g (p. 229)

- | | | | |
|---------------|-----------------|---------------------------|----------------------------|
| 1. 1 (twice) | 6. 3 (twice) | 11. -9 (twice) | 16. -5 (twice) |
| 2. 5 (twice) | 7. 4 (twice) | 12. 7 (twice) | 17. 6 (twice) |
| 3. 10 (twice) | 8. 9 (twice) | 13. 11 (twice) | 18. 20 (twice) |
| 4. -4 (twice) | 9. -1 (twice) | 14. -6 (twice) | 19. 8 (twice) |
| 5. -3 (twice) | 10. -10 (twice) | 15. $\frac{1}{2}$ (twice) | 20. $-\frac{2}{3}$ (twice) |

Exercise 13h (p. 230)

- | | | | |
|--------------------------|---------------------------------------|--|-----------------------|
| 1. $\frac{1}{2}$ and 2 | 9. $-\frac{2}{3}$ and -3 | 17. $-\frac{5}{6}$ and $2\frac{1}{2}$ | 25. $\pm\frac{12}{5}$ |
| 2. $1\frac{1}{2}$ and 4 | 10. $-\frac{2}{5}$ and -5 | 18. $-\frac{1}{2}$ and $-1\frac{1}{2}$ | 26. $\pm\frac{2}{3}$ |
| 3. $2\frac{1}{2}$ and 4 | 11. $-\frac{1}{2}$ and $\frac{2}{3}$ | 19. $-\frac{2}{3}$ and $-\frac{3}{4}$ | 27. $\pm\frac{5}{9}$ |
| 4. -1 and $-\frac{2}{3}$ | 12. $\frac{2}{5}$ and $-1\frac{1}{3}$ | 20. $3\frac{1}{2}$ and $-\frac{3}{5}$ | 28. $\pm\frac{2}{5}$ |
| 5. -7 and $2\frac{1}{2}$ | 13. $\frac{1}{3}$ and $\frac{1}{4}$ | 21. $\pm\frac{5}{4}$ | 29. $\pm\frac{5}{6}$ |
| 6. $\frac{2}{3}$ and 3 | 14. $-\frac{1}{3}$ and $2\frac{1}{2}$ | 22. $\pm\frac{9}{10}$ | 30. $\pm\frac{9}{2}$ |
| 7. $\frac{1}{3}$ and 2 | 15. $-\frac{1}{5}$ and $-\frac{3}{4}$ | 23. $\pm\frac{5}{2}$ | |
| 8. $1\frac{1}{2}$ and -4 | 16. $\frac{3}{4}$ and $1\frac{1}{2}$ | 24. $\pm\frac{4}{3}$ | |

Exercise 13i (p. 231)

- | | | | |
|-------------|-------------|--------------|-------------------------|
| 1. -5 and 6 | 2. -2 and 8 | 3. 3 and -12 | 4. $\frac{2}{3}$ and -2 |
|-------------|-------------|--------------|-------------------------|

- | | | | |
|--------------------------------------|--------------------------------------|-----------------------------|---------------------------------------|
| 5. 3 and -2 | 13. -2 and 5 | 21. 2 and 6 | 29. $\frac{1}{2}$ and -3 |
| 6. 1 and -7 | 14. 2 and 4 | 22. 4 and 5 | 30. 0 and 3 |
| 7. $\frac{1}{2}$ and -3 | 15. $\frac{1}{2}$ and $-\frac{1}{3}$ | 23. 5 and 7 | 31. 1 and 2 |
| 8. 3 and $-\frac{3}{5}$ | 16. $\frac{1}{3}$ and 4 | 24. 3 and 5 | 32. -1 and -2 |
| 9. -2 and 4 | 17. 2 and 5 | 25. 0 and $\frac{1}{2}$ | 33. $\frac{1}{3}$ and 2 |
| 10. -4 and 6 | 18. 1 and 7 | 26. 2 and 3 | 34. $-\frac{1}{5}$ and $1\frac{1}{2}$ |
| 11. 5 and 7 | 19. 2 and 4 | 27. 2 and 6 | |
| 12. $-\frac{1}{5}$ or $1\frac{1}{2}$ | 20. 3 and 7 | 28. -1 and $-\frac{2}{3}$ | |

Exercise 13j (p. 233)

- | | | |
|--------------------------------------|--|---|
| 1. -4 and 5 | 18. $-\frac{1}{2}$ and $-1\frac{1}{3}$ | 35. 5 and -9 |
| 2. 2 (twice) | 19. 0 and $1\frac{3}{4}$ | 36. -2 and 7 |
| 3. $\pm\frac{1}{3}$ | 20. $\frac{1}{3}$ and $\frac{1}{4}$ | 37. 7 and -4 |
| 4. 0 and $-3\frac{1}{2}$ | 21. $\frac{1}{3}$ and $-2\frac{1}{2}$ | 38. 5 and -11 |
| 5. -1 and -12 | 22. $-\frac{1}{3}$ and 2 | 39. -4 and -5 |
| 6. $\pm\frac{1}{4}$ | 23. $-\frac{1}{2}$ and $-1\frac{1}{2}$ | 40. -4 and -5 |
| 7. 0 and 6 | 24. $\pm\frac{1}{2}$ | 41. 0, 1 and 2 |
| 8. -5 and 7 | 25. 3 and -4 | 42. 0, 3 and -4 |
| 9. 2 and $-3\frac{1}{3}$ | 26. 3 and -1 | 43. 0, 2 and $2\frac{1}{2}$ |
| 10. -3 (twice) | 27. $\frac{1}{2}$ and $-\frac{1}{3}$ | 44. 0, 1 and 1 |
| 11. 1 and -7 | 28. 1 and 4 | 45. 0, $-\frac{1}{2}$ and -4 |
| 12. $\pm\frac{2}{5}$ | 29. -3 and 8 | 46. 0, 6 and 7 |
| 13. $\pm 2\frac{1}{2}$ | 30. 5 and 7 | 47. 0, -2 and 5 |
| 14. -2 and -9 | 31. -2 and $\frac{2}{3}$ | 48. 0, 5 and $-2\frac{1}{3}$ |
| 15. $\frac{1}{2}$ and $-\frac{2}{3}$ | 32. $-\frac{1}{3}$ and 2 | 49. 0, $\frac{3}{2}$ and $-\frac{3}{2}$ |
| 16. 0 and $2\frac{1}{2}$ | 33. 5 and -10 | 50. 0, 2 and 4 |
| 17. 2 and $-\frac{1}{3}$ | 34. -11 and 8 | |

Exercise 13k (p. 235)

Above average candidates should find a great deal of satisfaction from this exercise.

- | | | |
|--|--------------|--------------|
| 1. -2 or 8 | 2. -2 or 7 | 3. -7 or 6 |
| 4. $x + (x^2 - 6) = 66$; $x = -9$ or 8; 58 marbles | | |
| 5. $x + x^2 = 56$; $x = -8$ or 7; Ahmed is 7 and his father is 49 | | |
| 6. $x + (x^2 + 2) = 44$; $x = -7$ or 6; Kathryn is 6 and her mother is 38 | | |
| 7. $x(x + 5) = 84$; $x = 7$ or -12 ; Peter is 7 | | |
| 8. $x(x - 4) = 140$; $x = 14$ or -10 ; Ann is 10 | | |

9. $x(x + 3) = 28$; $x = 4$ or -7 ; 4 cm by 7 cm
 10. $x(x + 5) = 66$; $x = -11$ or 6 ; 6 cm by 11 cm
 11. $\frac{1}{2}x \times \frac{1}{2}x = 25$; $x = \pm 10$; 5 cm
 12. a) $A = 20x \text{ m}^2$, $B = x^2 \text{ m}^2$, $C = 30x \text{ m}^2$
 13. b) $x^2 + 50x = 104$; $x = 2$ or -52 ; path is 2 m wide

Exercise 13l (p. 238)

There are other possible solutions to some of these equations. The given solutions are positive numbers but there may be negative ones as well.

Notice that, in some cases, as the value tested goes up the number obtained goes down and more testing may be needed.

These questions give useful practice in the use of the memory functions in calculators, but note that some working needs to be written down.

Some pupils may find it easier to cope with a more systematic method for finding the numbers to try; in this case, the interval bisection method (i.e. using the value halfway between the two previous values) can be used. This has the advantage that it can be programmed into a computer.

132 Short Programs for the Mathematics Classroom (published by Stanley Thornes (Publishers) Ltd) gives examples of programs for finding roots of equations.

- | | |
|----------------|---------------------------------|
| 1. 1.3 and 1.4 | 5. 1.4 and 1.5 |
| 2. 1.3 and 1.4 | 6. 11.2 and 11.3 or 0.7 and 0.8 |
| 3. 2.0 and 2.1 | 7. 3.1 and 3.2 |
| 4. 2.8 and 2.9 | 8. 2.6 and 2.7 |
-
- | | |
|---------------------------------------|---------------|
| 9. a) 3.16 and 3.17 | b) 3.2 |
| 10. a) 1.62 and 1.63 or 7.37 and 7.38 | b) 1.6 or 7.4 |
| 11. a) 1.21 and 1.22 or 7.83 and 7.84 | b) 1.2 or 7.8 |
| 12. a) 4.17 and 4.18 | b) 4.2 |

Exercise 13m (p. 240)

- | | | | | | |
|--------------------|---------------------------|--------|---------------------------------------|-------------------------------------|----------------------|
| 1. a) -10 | b) 0 | c) 8 | 5. a) $\frac{1}{2}$ and $\frac{4}{5}$ | b) $\frac{2}{5}$ and $-\frac{1}{3}$ | c) $\pm \frac{2}{3}$ |
| 2. a) 0 or -7 | b) 0 or $\frac{1}{2}$ | | 6. a) 0 and 2 | b) 0 and $\frac{3}{4}$ | |
| 3. a) 3 and 8 | b) 2 and $-\frac{3}{5}$ | | 7. a) 5 and -9 | b) 5 and -6 | |
| 4. a) 7 and -5 | b) 5 and 8 | | 8. $2.5, 2.6$ | | |

Exercise 13n (p. 240)

- | | | | |
|--|---------------------------------------|---------------------------------------|--------|
| 1. a) -2 | b) 0 | c) 12 | d) 0 |
| 2. a) 0 and 2 | b) 0 and $-\frac{3}{7}$ | | |
| 3. a) 2 and -5 | b) -2 and $1\frac{1}{3}$ | c) $-1\frac{1}{2}$ and $1\frac{1}{2}$ | |
| 4. a) -3 and 2 | b) -5 and -6 | | |
| 5. a) $\frac{1}{5}$ and $-\frac{3}{4}$ | b) $-\frac{2}{5}$ and $-2\frac{1}{3}$ | | |
| 6. a) 0 and $-\frac{2}{3}$ | b) 0 and $-\frac{3}{7}$ | | |
| 7. a) -4 and 8 | b) -2 and 4 | | |
| 8. $19.6, 19.7$ or $0.3, 0.4$ | | | |

Exercise 13p (p.240)

- | | | |
|------------------------------------|--------------------------------|--------------------------------|
| 1. a) -11 | b) 0 | c) 0 |
| 2. a) $0, -7$ | b) $0, \frac{3}{4}$ | |
| 3. a) $-4, 5$ | b) $1\frac{3}{4}, -3$ | c) $\frac{3}{5}, -\frac{3}{5}$ |
| 4. a) $5, -3$ | b) $-4, -8$ | |
| 5. a) $-\frac{1}{5}, -\frac{3}{4}$ | b) $-\frac{2}{7}, \frac{1}{4}$ | |
| 6. a) $0, -1\frac{1}{3}$ | b) $0, -1\frac{2}{3}$ | |
| 7. a) $-5, 2$ | b) $-10, 3$ | |
| 8. $1.3, 1.4$ | | |