CHAPTER 12 Algebraic Factors

Each type of factor could be introduced as the converse of an expansion from the previous chapter.

EXERCISE 12a (p.205)

We start with common factors which are often forgotten when factorising at a later date. Encourage multiplying out to check the results.

1. $4(x + 1)$
2. $3(4x - 1)$
3. $2(3a + 1)$
4. $5(a - 2b)$
5. $3(t - 3)$
6. $5(2a - 1)$
7. $4(3a + 1)$
8. $2(a + 2b)$
9. $7(2x - 1)$
10. $x(x + 2)$
11. $x(x - 7)$
12. $a(a + 6)$
13. $x(2x + 1)$
14. $2t(2 - t)$
15. $x(x + 5)$
16. $x(x - 4)$
17. $b(b + 4)$
18. $a(4a - 1)$
19. $2x(x - 3)$
20. $2z(z^2 + 2)$

21. $5a(5a - 1)$
22. $4x(3x + 4)$
23. $5b(a - 2c)$
24. $3y(y + 9)$
25. $2a(a - 6)$
26. $2p(3p + 1)$
27. $3y(3y - 2)$
28. $2(x^2 + 2x + 3)$
29. $5(2a^2 - a + 4)$
30. $b(a + 4c - 3d)$
31. $4(2x - y + 3z)$
32. $3a(3b - 2c - d)$
33. $3(x^2 - 2x + 3)$
34. $4(a^2 + 2a - 1)$
35. $x(5y + 4z + 3)$
36. $5b(a + 2c + d)$
37. $2y(x - 2z + 4w)$
38. $x^2(x + 1)$
39. $x^2(1 - x)$
40. $5a^2(4 - a)$

41. $4x^2(3x - 4)$
42. $4x^2(x^2 + 3)$
43. $a^2(1 + a)$
44. $b^2(b - 1)$
45. $2x^2(2x - 1)$
46. $9a^2(3 - 2a)$
47. $5x^2(2 - 3x^2)$
48. $4(3x + 2)$
49. $4x(2x + 3)$
50. $3(3x^2 - 2x + 4)$
51. $5x(x^2 - 2)$
52. $4q(2p + r)$
53. $x(x - 8)$
54. $3(4 + 3y^2)$
55. $4x(3y + 4z + 2)$
56. $2x(2x^2 + 3)$
57. $4bc(3a - 2d)$
58. $\frac{1}{2}h(a + b)$
59. $m(g - a)$
60. $\frac{1}{2}m(v^2 + u^2)$

61. $P\left(1 + \frac{RT}{100}\right)$
62. $\pi r(2r + h)$
63. $\pi(R^2 + r^2)$
64. $2g(h_1 - h_2)$
65. $m(\frac{1}{2}v^2 - gh)$
66. $\frac{\pi r^2}{3}(4r - h)$

67. $\pi r(3r + 2h)$ **68.** $\frac{1}{2} mu(u + 1)$ **69.** $\frac{1}{4} c(2b - a)$

EXERCISE 12b (p. 208)

It is important to point out that it does not matter which bracket is written first, i.e. $(x + 2)(x + 3)$ is identical to $(x + 3)(x + 2)$.

1. $(x + 1)(x + 2)$
2. $(x + 1)(x + 5)$
3. $(x + 3)(x + 4)$
4. $(x + 3)(x + 5)$
5. $(x + 1)(x + 20)$
6. $(x + 1)(x + 7)$
7. $(x + 6)(x + 2)$
8. $(x + 1)(x + 12)$
9. $(x + 1)(x + 15)$
10. $(x + 2)(x + 10)$
11. $(x + 4)(x + 4)$
12. $(x + 3)(x + 12)$
13. $(x + 1)(x + 18)$
14. $(x + 2)(x + 20)$
15. $(x + 1)(x + 8)$
16. $(x + 3)(x + 3)$
17. $(x + 2)(x + 18)$
18. $(x + 3)(x + 6)$
19. $(x + 5)(x + 6)$
20. $(x + 4)(x + 10)$

EXERCISE 12c (p. 208)

1. $(x - 1)(x - 8)$
2. $(x - 3)(x - 4)$
3. $(x - 2)(x - 15)$
4. $(x - 4)(x - 7)$
5. $(x - 6)(x - 7)$
6. $(x - 2)(x - 3)$
7. $(x - 1)(x - 15)$
8. $(x - 3)(x - 3)$
9. $(x - 2)(x - 16)$
10. $(x - 7)(x - 9)$

EXERCISE 12d (p. 210)

1. $(x + 2)(x - 3)$
2. $(x + 5)(x - 4)$
3. $(x - 4)(x + 3)$
4. $(x - 4)(x + 7)$
5. $(x + 5)(x - 3)$
6. $(x - 6)(x + 4)$
7. $(x - 3)(x + 9)$
8. $(x - 11)(x + 2)$
9. $(x - 7)(x + 5)$
10. $(x - 10)(x + 2)$

EXERCISE 12e (p. 211)

To some, the worked example may appear to be too detailed. Most pupils require a definite plan of attack and will find the given method very helpful until they feel confident enough to go straight to the answer.

1. $(x + 2)(x + 7)$
2. $(x - 3)(x - 7)$
3. $(x + 7)(x - 2)$
4. $(x + 6)(x - 5)$
5. $(x + 1)(x + 8)$
6. $(x - 5)(x - 5)$
7. $(x + 9)(x - 1)$
8. $(x - 13)(x - 2)$
9. $(x + 8)(x - 7)$
10. $(x + 2)(x + 30)$
11. $(x + 3)(x - 9)$
12. $(x + 20)(x - 4)$
13. $(x + 1)(x + 13)$
14. $(x - 2)(x + 14)$
15. $(x + 10)(x - 8)$
16. $(x - 5)(x - 6)$
17. $(x - 4)(x + 12)$
18. $(x + 6)(x + 12)$
19. $(x + 4)(x + 13)$
20. $(x + 2)(x - 14)$
21. $(x + 3)(x + 8)$
22. $(x + 3)(x - 14)$
23. $(x - 2)(x - 16)$
24. $(x + 12)(x - 5)$

EXERCISE 12f (p. 211)

1. $(x + 1)(x + 8)$
2. $(x - 3)(x - 3)$
3. $(x + 4)(x + 7)$
4. $(4 - x)(5 + x)$
5. $(x + 3)(x + 3)$
6. $(x - 1)(x - 8)$
7. $(x + 2)(x + 15)$
8. $(9 + x)(3 - x)$
9. $(x + 2)(x + 11)$

10. $(x - 13)(x + 2)$
11. $(x - 1)(x - 7)$
12. $(x - 6)(x + 7)$
13. $(x - 8)(x + 3)$
14. $(x - 2)(x - 7)$
15. $(x + 1)(x + 27)$
16. $(x - 7)(x + 9)$
17. $(x + 5)^2$
18. $(x - 5)^2$
19. $(x + 2)^2$
20. $(x - 7)^2$
21. $(x + 6)^2$
22. $(x - 6)^2$
23. $(x - 2)^2$
24. $(x + 8)^2$

EXERCISE 12g (p. 212)

Many pupils need much convincing that $6 - 5x - x^2$ and $x^2 + 5x - 6$ do not factorise to give the same answers. The problem is not helped later when $6 - 5x - x^2 = 0$ becomes $x^2 + 5x - 6 = 0$. Time spent on distinguishing between an expression and an equation, i.e. on the difference between factorising an expression and using factors to solve an equation, will be time well spent.

1. $(2 + x)(1 - x)$
2. $(3 - x)(2 + x)$
3. $(1 - x)(4 + x)$
4. $(4 - x)(2 + x)$
5. $(3 + x)(2 - x)$
6. $(2 - x)(1 + x)$
7. $(4 + x)(2 - x)$
8. $(5 + x)(1 - x)$
9. $(5 + x)(2 - x)$
10. $(6 - x)(2 + x)$
11. $(5 - x)(1 + x)$
12. $(7 + x)(2 - x)$
13. $(6 - x)(1 + x)$
14. $(5 + x)(4 - x)$
15. $(5 + x)(3 - x)$
16. $(4 - x)(3 + x)$

EXERCISE 12h (p. 213)

1. $(x + 5)(x - 5)$
2. $(x + 2)(x - 2)$
3. $(x + 10)(x - 10)$
4. $(x + 1)(x - 1)$
5. $(x + 8)(x - 8)$
6. $(x + 4)(x - 4)$
7. $(x + 6)(x - 6)$
8. $(x + 9)(x - 9)$
9. $(x + 7)(x - 7)$
10. $(3 + x)(3 - x)$
11. $(6 + x)(6 - x)$
12. $(10 + x)(10 - x)$
13. $(a + b)(a - b)$
14. $(3y + z)(3y - z)$
15. $(4 + x)(4 - x)$
16. $(5 + x)(5 - x)$
17. $(9 + x)(9 - x)$
18. $(x - y)(x - y)$

EXERCISE 12i (p.214)

1. $3(x + 4)$
2. $5x(5x + 2)$
3. $4(3x^2 - 2)$
4. $7(2x + 3)$
5. $2(2x^2 + 1)$
6. $7(3x - 1)$
7. $9x(x - 2)$
8. $4(5x + 3)$
9. $2(2x - 7)$
10. $4x(2x - 1)$
11. $2(x + 3)(x + 4)$
12. $3(x - 1)(x - 8)$
13. $7(x + 1)^2$
14. $4(x + 3)(x - 4)$
15. $5(x + 1)(x + 7)$
16. $3(x + 2)(x + 6)$
17. $4(x - 3)^2$
18. $5(x + 2)(x - 3)$
19. $2(x + 2)(x - 11)$
20. $3(x - 5)(x + 8)$

EXERCISE 12j (p. 215)

1. $(2x + 1)(x + 1)$
2. $(3x - 2)(x - 1)$
3. $(4x + 3)(x + 1)$
4. $(2x - 1)(x - 3)$
5. $(3x + 1)(x + 4)$
6. $(3x - 2)(x - 2)$
7. $(2x + 1)(x + 4)$
8. $(5x - 2)(x - 3)$
9. $(2x + 3)(x + 4)$
10. $(7x - 1)(x - 4)$
11. $(2x + 1)(x - 2)$
12. $(3x + 4)(x - 1)$
13. $(5x + 2)(x - 3)$
14. $(x + 2)(4x - 3)$
15. $(3x - 2)(x + 4)$
16. $(7x + 2)(x - 3)$
17. $(6x + 5)(x - 2)$
18. $(5x - 4)(x - 3)$
19. $(3x + 4)(x - 5)$
20. $(4x - 3)(x + 5)$

EXERCISE 12k (p. 216)

1. $(3x + 2)(2x + 1)$
2. $(2x + 3)(3x + 5)$
3. $(3x + 1)(5x + 2)$
4. $(2x + 3)(6x + 5)$
5. $(7x + 2)(5x + 2)$
6. $(3x – 1)(2x – 3)$
7. $(3x – 2)(3x – 4)$
8. $(2x – 1)(8x – 1)$
9. $(5x – 3)(3x – 7)$
10. $(5x – 2)(4x – 3)$
11. $(4x + 1)(2x – 3)$
12. $(5x – 2)(3x + 1)$
13. $(3x + 2)(7x – 4)$
14. $(10x + 3)(8x – 3)$
15. $(3x + 4)(8x – 5)$
16. $(3a – 5)(2a + 3)$
17. $(3t – 2)(2t + 1)$
18. $(3b – 2)^2$
19. $(x – 2y)(5x + 3y)$
20. $(x – 2)(4x – 3)$

EXERCISE 12l (p. 217)

1. $(2x + 5)(2x – 5)$
2. $(3x + 2)(3x – 2)$
3. $(6a + 1)(6a – 1)$
4. $(4a + b)(4a – b)$
5. $(3x + 5)(3x – 5)$
6. $(2a + 1)(2a – 1)$
7. $(4a + 3b)(4a – 3b)$
8. $(5s + 3t)(5s – 3t)$
9. $(10x + 7y)(10x – 7y)$
10. $(3y + 4z)(3y – 4z)$
11. $(2x + 7y)(2x – 7y)$
12. $(9x + 10y)(9x – 10y)$
13. $(3a + 2b)(3a – 2b)$
14. $(8p + 9q)(8p – 9q)$
15. $3(a + 3b)(a – 3b)$
16. $2(3t + 5s)(3t – 5s)$
17. $3(3x + y)(3x – y)$
18. $5(3x + 2)(3x – 2)$
19. $5(a + 2)(a – 2)$
20. $5(3 + b)(3 – b)$

21. $\frac{1}{2}(a + 2b)(a – 2b)$

22. $\left(\frac{a}{2} + \frac{b}{3}\right)\left(\frac{a}{2} – \frac{b}{3}\right)$ or $\frac{1}{36}(3a + 2b)(3a – 2b)$

23. $\frac{1}{3}(9x + y)(9x – y)$

24. $\left(\frac{x}{4} + \frac{y}{5}\right)\left(\frac{x}{4} – \frac{y}{5}\right)$ or $\frac{1}{400}(5x + 4y)(5x – 4y)$

EXERCISE 12m (p. 218)

1. 7.5
2. 18.5
3. 17.7
4. 35.04
5. 31.2
6. 20.4
7. 12.9
8. 178.97
9. 1000
10. 336
11. 53.2
12. 5.336
13. 8
14. 140
15. 75.8
16. 0.526

EXERCISE 12n (p. 218)

This is an important exercise. Forgetting to extract a common factor results in many expressions being more difficult to factorise than they need be.

1. $5(x + 1)(3x + 2)$
2. $2(x – 2)(2x + 1)$
3. $3(x + 1)(2x + 1)$
4. $3(x – 2)(6x + 5)$
5. $2(x + 5)(4x – 3)$
6. $2(x + 1)(4x + 3)$
7. $5(x – 3)(5x + 2)$
8. $3(x – 1)(3x + 4)$
9. $2(x + 4)(3x + 1)$
10. $5(x + 4)(3x – 2)$
11. $2(3x – 2)(3x – 4)$
12. $3(2x – 1)(8x – 1)$
13. $2(2x + 1)(3x + 2)$
14. $5(4x – 3)(5x – 2)$
15. $4(2x + 1)(3x – 2)$
16. $7(x + 4)(3x – 2)$
17. $(4 + 3x)(1 – 2x)$
18. $(4 – 3x)(3 + 4x)$
19. $(7 – x)(3 + 4x)$
20. $2(2 – x)(6 – x)$
21. $2(4 + x)(2 – 3x)$
22. $(9 – x)(1 + x)$
23. $(12 + x)(1 – x)$
24. $2(2 + 3x)^2$
25. $5(3 – x)^2$
26. $5(2 + x)(2 + 3x)$

EXERCISE 12p (p. 219)

1. $(x + 5)(x + 8)$
2. $(3x + 1)(2x + 1)$
3. $(x + 6)(x - 6)$
4. Does not factorise
5. $(x - 2)(x - 6)$
6. $(2x - 3)(x + 5)$
7. $(x + 7)(x - 1)$
8. $(5x - 2)(x + 1)$
9. $(x - 3)(x - 8)$
10. $(3x + 2)(x + 3)$
11. $(x + 15)(x - 1)$
12. $(4x - 1)(3x - 1)$
13. $(x + 2)(x + 6)$
14. $(4x + 1)(2x - 1)$
15. $(x + 7)(x - 7)$
16. Does not factorise
17. $(3x + 2)(2x - 5)$
18. $(x + 6)(x + 7)$
19. $(2x + 3y)(2x - 3y)$
20. $(5x - 4)(3x - 2)$
21. $(2x - 3)(3x + 2)$
22. $(x + 13)(x - 2)$
23. $2(3x + 1)(5x - 2)$
24. $(4 + x)(7 - x)$
25. $(2x - 1)(3x + 4)$
26. $5(2x + 1)(3x + 2)$
27. $(x + 2)(x + 9)$
28. $(x - 4)(x - 6)$
29. $4(x + 2y)(x - 2y)$
30. Does not factorise
31. $2(3x + 2)(2x - 5)$
32. $(x - 2)(x + 15)$
33. $(2 - x)(14 + x)$
34. $(a - 7)(a - 9)$
35. $2(3 - 2x)(1 - 2x)$
36. $(1 + 2x)(1 + 4x^2)$
37. $(x + 17)(x - 4)$
38. $(2x - 1)(x^3 + 2)$
39. $3(2x + 1)(x - 2)$
40. $(p + 1)(p^2 + 1)$
41. $(a + b + c)(a + b - c)$
42. $(29x + 1)(4x - 1)$
43. $(a + 16)(a + 7)$
44. $(x^2 + y + 1)(x^2 - y - 1)$
45. $(a - 8)(3a - 7)$
46. $2(x + 7)(x - 11)$
47. $(2x + y - z)(2x - y + z)$
48. $(ab + 18)(ab - 19)$

EXERCISE 12q (p. 220)

1. a) $7a + 21$ b) $3x - 6y$
2. a) $x^2 + 14x + 40$ b) $6x^2 - 19x + 15$
3. a) $25 + 10x + x^2$ b) $25 - 10x + x^2$ c) $25 - x^2$
4. a) $10(a + 2)$ b) $5p(3p - 2)$
5. a) $(a + 1)(a^2 + 1)$ b) $(k + l)(2m - n)$
6. a) $(x - 3)(x + 9)$ b) $(x - 7)(5x - 7)$ c) $\left(a + \frac{b}{2}\right)\left(a - \frac{b}{2}\right)$
7. a) $(5x + 2)(2x - 3)$ b) $(10a + 9b)(10a - 9b)$

EXERCISE 12r (p. 221)

1. a) $5a^2 + 15a$ b) $12x^2 - 8xy$
2. a) $y^2 - 9y + 20$ b) $15x^2 - 14xy - 8y^2$
3. a) $4p^2 + 12pq + 9q^2$ b) $4p^2 - 12pq + 9q^2$ c) $4p^2 - 9q^2$
4. a) $4z^2(2z - 1)$ b) $5y(x - 4z)$
5. a) $(m + 1)(2 + 3n)$ b) $(a + 2b)(c - 2d)$
6. a) $(x + 3)(x - 9)$ b) $(4x - 1)(x + 7)$ c) $(2m + 9n)(2m - 9n)$
7. a) $3(x - 3)(5x - 3)$ b) Does not factorise or $5(3 + 5x - 4x^2)$

EXERCISE 12s (p.221)

1. a) $4a + 28$ b) $6x^2 - 9xy$

2. a) $x^2 + 12x + 27$ b) $15x^2 - x - 2$

3. a) $25x^2 + 20x + 4$ b) $25x^2 - 20x + 4$ c) $25x^2 - 4$

4. a) $6z(2z - 1)$ b) $4y(2x - 3z)$

5. a) $(z + 2)(z^2 + 1)$ b) $(3a + b)(c + 2)$

6. a) $(x - 6)(x + 4)$ b) $(2a + 5)(2a - 3)$ c) $\left(3m + \frac{n}{3}\right)\left(3m - \frac{n}{3}\right)$

7. a) $(5x - 3)(3x + 2)$ b) $(3 + 5x)(2 - 3x)$