

CHAPTER 10 Angles in Circles

In Exercise 10a a movement starts towards a more formal and non-numerical treatment of geometry. The first section revises the basic facts and begins the progression towards a reasoned answer. The proof that the sum of the interior angles of a triangle is 180° can be used to demonstrate what can be reasoned from a few known facts. It can also be used to demonstrate what is acceptable as a reason. However the emphasis throughout this book is still on the intuitive recognition of facts.

Geometry is put on a more formal and non-numerical basis in Book 4A with an introduction to the deductive nature of Euclidian Geometry.

EXERCISE 10a (p. 163)

Numbers 15–20 are non-numerical and several examples should be used for discussion before pupils attempt them on their own. It is sensible to accept, as reason, any facts that they know, e.g. in Number 4 they may use the fact that opposite angles of a parallelogram are equal. Some discussion is also necessary on naming angles when there is more than one angle at a vertex. Either the angle must be clearly marked on the diagram with a small letter or three letters have to be used.

- | | |
|----------------|--|
| 1. 70° | 8. 66° |
| 2. 110° | 9. $d = 75^\circ, e = 65^\circ, f = 140^\circ$ |
| 3. 60° | 10. $p = 60^\circ, q = 60^\circ, r = 120^\circ, s = 60^\circ$ |
| 4. 70° | 11. $k = 30^\circ, l = 30^\circ, m = 30^\circ, n = 60^\circ$ |
| 5. 45° | 12. $g = 24^\circ, h = 156^\circ, i = 74^\circ$ |
| 6. 55° | 13. $w = 73^\circ, x = 34^\circ, y = 34^\circ, z = 73^\circ$ |
| 7. 125° | 14. $d = 64^\circ, e = 64^\circ, f = 116^\circ, g = 116^\circ, h = 64^\circ$ |

EXERCISE 10b (p. 168)

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|--------|--------|--------|--------|---------|
| 1. Yes | 3. Yes | 5. Yes | 7. Yes | 9. Yes |
| 2. No | 4. No | 6. No | 8. No | 10. Yes |

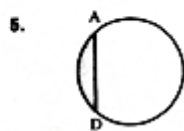
EXERCISE 10c (p. 170)

- Reflection in x -axis; Yes
- Rotation of 90° anticlockwise about 0; Yes
- Enlargement, scale factor 2, centre $(-4, 0)$; No
- Translation $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$; Yes
- Reflection in y -axis; Yes
- Enlargement, scale factor $\frac{1}{2}$, centre $(0, 0)$; No
- Rotation of 90° clockwise about $(5, 2)$; Yes
- Translation $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$; Yes

Many pupils have problems with circle questions because they do not have a clear understanding of a) the meaning of “subtends”, b) what a segment of a circle is. The next two exercises attempt to remedy this.

EXERCISE 10d (p. 174)

1. AB, AC, AD, BC, BD, CD, Yes, AC.

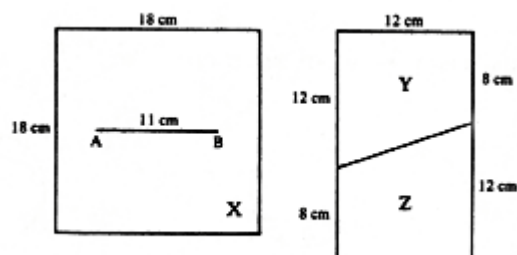


EXERCISE 10e (p. 176)

- | | |
|-----------------------------|---|
| 1. Minor arc DC | 7. Minor arc BE |
| 2. Minor arc BC | 8. Minor arc CD |
| 3. $\angle ACB, \angle ADB$ | 9. CE |
| 4. $\angle BAC, \angle BDC$ | 10. DB |
| 5. DA | 11. a) $\angle ACB, \angle AEB$ b) $\angle BAC, \angle BEC$ |
| 6. AB | 12. a) $\angle ABE, \angle ACE, \angle ADE$ b) $\angle CDE, \angle CAE, \angle CBE$ |

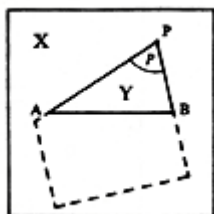
Experimental Work Some teachers (and pupils!) may find the experimental work detailed below quite useful. The work may be illustrated by the teacher or, better still, pupils should be encouraged to make their own models using, for example, covers from old exercise books. As an introduction you may choose to go through all the experiments (results) at one sitting, but to repeat them as each new fact is taught. Four experiments are listed, three of them yielding the five facts used in this chapter. The alternate segment result will not be used until Book 4A, but it is certainly worth a mention here since the model illustrates it so nicely. All the results can be confirmed by diagrams and measurements.

Preparing the Model You require two pieces of cardboard or stiff paper, preferably of different colours. The first should be a square of side approximately 18 cm and the other a rectangle measuring 20 cm by 12 cm.

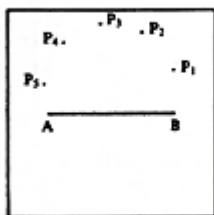


In the square piece cut a slot AB, 11 cm, long, near the middle of the card. Cut the second piece along the line shown in the diagram to give two identical trapeziums Y and Z.

Experiment 1: (illustrating “angles in the same segment” and “opposite angles of a cyclic quadrilateral” result).

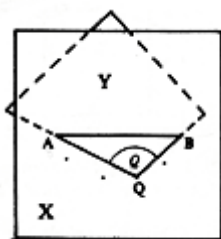


Take the trapezium Y and mark the acute angle P. Push P through the slot AB from behind, until the trapezium will not go any further. Mark with a dot the position of P on the square X. Rotate the card Y into another position (again making sure that Y fills the slot) and mark the new position of P. Do this several times marking each position of P as shown below.



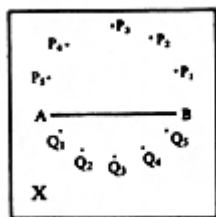
Now mark the obtuse angle of Y with the letter Q.

Push Y through the slot from behind as shown in the diagram and mark the position of Q. Rotate Y to give several positions of Q.



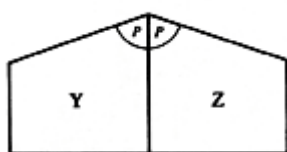
Take Y out and note that all the marked points look as though they lie on a circle. The different positions of P seem to lie on a major arc and the different positions of Q on a minor arc.

Card X should now look like this:

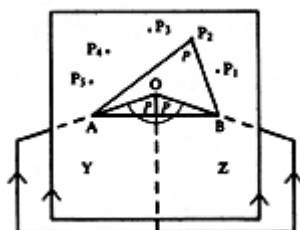


Experiment 2: (illustrating “angle at centre” result).

Take Y and Z and place them together as shown below (you might find it useful to sellotape them together).



Now push them together through the slot AB from behind, keeping the edges of Y and Z parallel to the edges of the rectangle as indicated. Mark O as shown.

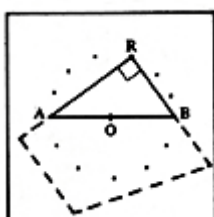


Take Y and Z out of the slot. With centre O and radius OA draw the major arc AB which will be seen to pass through P_1, P_2, \dots

Experiment 3: (illustrating “angle in a semicircle” result).

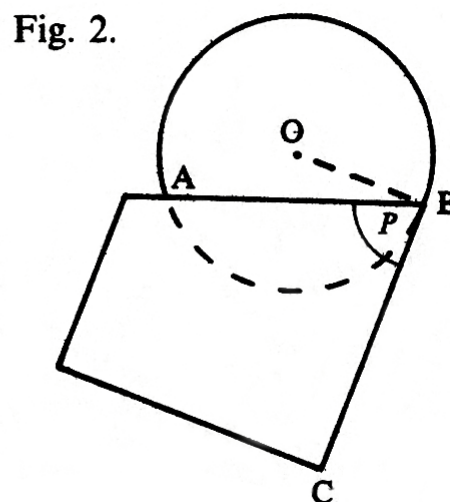
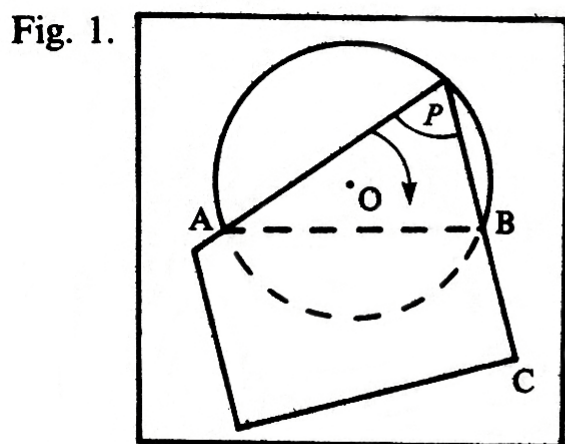
Turn the square card over to use the opposite side. Use one of the right angles from Y or Z and place it upwards through the slot AB from behind. Mark the position of the right angle R. Repeat this with R in several different positions.

Now place the right angle downwards through AB from behind and mark several additional positions of R.



Remove the trapezium and find the midpoint O of AB. Centre O, radius OA, draw a circle.

Experiment 4: (“alternate segment” result).



Place one trapezium on the circle obtained in the first experiment such that it gives one position for P. This is shown in Figure 1. Now rotate the trapezium downwards about A so that P moves along the circle towards B. Figure 2 shows the position when AP coincides with AB. OB is a radius and you can see that the side CB on the trapezium has only one point of contact with the circle, that is at B.

CB is a tangent to the circle at B.

Since $\angle APB = \angle ABC$ this experiment shows that:

the angle between a tangent and a chord drawn at the point of contact, is equal to the angle in the alternate segment.

Exercise 10f (p. 176)

Answers depend on pupils’ drawings but in each question the angles should be equal.

Exercise 10g (p. 177)

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|---------------------------------|----------------------------------|----------------------------------|
| 1. $h = 38^\circ$ | 4. $p = 72^\circ, q = 57^\circ$ | 7. $w = 57^\circ, x = 123^\circ$ |
| 2. $i = 39^\circ, j = 46^\circ$ | 5. $l = 100^\circ$ | 8. $c = 114^\circ$ |
| 3. $x = 33^\circ = y$ | 6. $x = 108^\circ, y = 26^\circ$ | |

Exercise 10h (p. 179)

Answers depend on pupils’ drawings but in each question $y = 2x$.

Exercise 10i (p. 180)

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|-------------------|--------------------|-------------------|--------------------|
| 1. $d = 80^\circ$ | 3. $f = 114^\circ$ | 5. $g = 98^\circ$ | 7. $l = 132^\circ$ |
| 2. $e = 64^\circ$ | 4. $i = 38^\circ$ | 6. $h = 32^\circ$ | 8. $m = 102^\circ$ |

Exercise 10j (p. 182)

Answers depend on pupils’ drawings but in each question $p + q = 180^\circ$.

Exercise 10k (p. 183)

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|--------------------|----------------------------------|----------------------------------|
| 1. $d = 108^\circ$ | 4. $k = 115^\circ$ | 7. $g = 121^\circ, h = 68^\circ$ |
| 2. $e = 84^\circ$ | 5. $l = 131^\circ$ | 8. $i = 110^\circ, j = 50^\circ$ |
| 3. $f = 103^\circ$ | 6. $m = 87^\circ, n = 112^\circ$ | |

Exercise 10l (p. 185)

Answers depend on pupils’ drawings but the conclusions should be:

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|------------|--------------------|----------------------------------|
| 1. $p = q$ | 4. $x = w$ | 7. $p = 54^\circ, q = 76^\circ$ |
| 2. $v = u$ | 5. $y = 100^\circ$ | 8. $r = 126^\circ, s = 83^\circ$ |
| 3. $s = r$ | 6. $z = 109^\circ$ | |

Exercise 10m (p. 186)

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|--|---|
| 1. $a = 44^\circ$ | 8. $r = 110^\circ, s = 122^\circ$ |
| 2. $c = 60^\circ, d = 46^\circ$ | 9. $x = 30^\circ, y = 58^\circ, z = 88^\circ$ |
| 3. $g = 116^\circ$ | 10. $c = 25^\circ, d = 25^\circ, e = 50^\circ$ |
| 4. $b = 78^\circ$ | 11. $h = 116^\circ, i = 32^\circ$ |
| 5. $e = 34^\circ, f = 52^\circ$ | 12. $l = 126^\circ, m = 63^\circ, n = 117^\circ$ |
| 6. $h = 72^\circ$ | 13. $u = 34^\circ, v = 68^\circ, w = 56^\circ, x = 56^\circ$ |
| 7. $l = 154^\circ, m = 40^\circ, n = 37^\circ$ | 14. $k = 62^\circ, l = 56^\circ, m = 124^\circ, n = 16^\circ$ |

Exercise 10n (p. 187)

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|-----------------------|-----------------------|-----------------------|
| 1. $d = 90^\circ = e$ | 2. $f = 90^\circ = g$ | 3. $h = 90^\circ = i$ |
|-----------------------|-----------------------|-----------------------|

Exercise 10p (p. 188)

1. $d = 90^\circ, e = 52^\circ$
2. $f = 90^\circ, g = 45^\circ$
3. $h = 90^\circ, i = 26^\circ$
4. $l = 90^\circ, m = 61^\circ$
5. $j = 90^\circ, k = 55^\circ$
6. $p = 90^\circ, q = 38^\circ$
7. $r = 90^\circ, s = 52^\circ, t = 90^\circ, u = 43^\circ$
8. $d = 90^\circ, e = 45^\circ, f = 90^\circ, g = 18^\circ$
9. $c = 90^\circ, d = 58^\circ, e = 32^\circ$
10. $v = 90^\circ, w = 47^\circ, x = 90^\circ, y = 51^\circ$
11. $j = 90^\circ, k = 33^\circ, l = 33^\circ, m = 57^\circ$
12. $f = 45^\circ, g = 58^\circ, h = 45^\circ, i = 32^\circ$

Exercise 10q (p. 190)

1. $d = 106^\circ$
2. $e = f = 38^\circ$
3. $d = 34^\circ, e = 68^\circ$
4. $x = 75^\circ, y = 15^\circ, z = 132^\circ$
5. $p = 36^\circ = q, r = 39^\circ$
6. $x = 112^\circ, y = 68^\circ, z = 112^\circ$
7. $g = 54^\circ, h = 120^\circ$
8. $d = 37^\circ, e = 53^\circ, f = 57^\circ, g = 33^\circ$