

CHAPTER 20 Vectors

This unit is optional. It can be done later (it is repeated with different exercises in Book 3) or omitted completely. If a brief introduction is thought appropriate, Exercise 20a and Exercise 20b form a good start.

Some pupils may suggest the need to state a time in the initial paragraph (p. 294); this can be dealt with if it arises but need not be introduced otherwise.

EXERCISE 20a (p. 294)

1. scalar 2. vector 3. scalar 4. scalar 5. vector

EXERCISE 20b (p. 295)

$$\begin{array}{ll}
 1. \begin{pmatrix} 3 \\ 2 \end{pmatrix} & 4. \begin{pmatrix} -2 \\ 2 \end{pmatrix} \\
 2. \begin{pmatrix} 4 \\ 1 \end{pmatrix} & 5. \begin{pmatrix} -3 \\ 4 \end{pmatrix} \\
 3. \begin{pmatrix} 4 \\ 0 \end{pmatrix} & 6. \begin{pmatrix} -5 \\ -3 \end{pmatrix}
 \end{array}$$

$$\begin{array}{l}
 7. \mathbf{g} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \quad \mathbf{h} = \begin{pmatrix} -4 \\ 0 \end{pmatrix} \quad \mathbf{i} = \begin{pmatrix} 6 \\ 2 \end{pmatrix} \\
 \mathbf{j} = \begin{pmatrix} -6 \\ 7 \end{pmatrix} \quad \mathbf{k} = \begin{pmatrix} -6 \\ -2 \end{pmatrix} \quad \mathbf{l} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \\
 \mathbf{m} = \begin{pmatrix} 0 \\ -4 \end{pmatrix} \quad \mathbf{n} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}
 \end{array}$$

EXERCISE 20c (P. 297)

1. (7,4) 6. (8,0) 11. (-9,-1) 15. (3,-2) 19. (-7,4)
 2. (1,-2) 7. (-1,0) 12. (-7,-3) 16. (-2,-3) 20. (-1,-10)
 3. (-3,7) 8. (-9,-8) 13. (-6,-1) 17. (1,-3) 21. (-6,-6)
 4. (1,-5) 9. (2,0) 14. (-2,-3) 18. (1,5) 22. (-1,10)
 5. (8,1) 10. (7,-4)

EXERCISE 20d (p. 299)

$$\begin{array}{llll}
 1. \begin{pmatrix} 6 \\ 2 \end{pmatrix} & 3. \begin{pmatrix} -6 \\ -1 \end{pmatrix} & 5. \begin{pmatrix} -5 \\ -6 \end{pmatrix} & 7. \begin{pmatrix} -2 \\ -2 \end{pmatrix} & 9. \begin{pmatrix} 0 \\ -12 \end{pmatrix} \\
 2. \begin{pmatrix} 5 \\ -1 \end{pmatrix} & 4. \begin{pmatrix} 6 \\ 5 \end{pmatrix} & 6. \begin{pmatrix} 2 \\ -2 \end{pmatrix} & 8. \begin{pmatrix} -4 \\ -5 \end{pmatrix} & 10. \begin{pmatrix} 2 \\ 8 \end{pmatrix}
 \end{array}$$

EXERCISE 20e (p. 301)

1. a) $b = 2a$ b) $c = -a$ c) $d = 3a$ d) $e = a$ e) $b = 2e$ f) $d = -3c$
2. $a = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ $b = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$ $c = \begin{pmatrix} -4 \\ -6 \end{pmatrix}$ $d = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $e = \begin{pmatrix} 8 \\ -4 \end{pmatrix}$ $f = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$ $g = \begin{pmatrix} 6 \\ 9 \end{pmatrix}$ $h = \begin{pmatrix} -8 \\ 4 \end{pmatrix}$
- $e = 2a$, $f = -a$, $h = -2a$, $c = 2b$, $d = -b$, $g = -3b$, $h = -e$, $g = 3d$, $h = 2f$, ...

3. $\begin{pmatrix} 8 \\ 12 \end{pmatrix}, \begin{pmatrix} -4 \\ -6 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

4. $\begin{pmatrix} 2 \\ -4 \end{pmatrix}, \begin{pmatrix} -4 \\ 8 \end{pmatrix}, \begin{pmatrix} 4 \\ -8 \end{pmatrix}$

5. $\begin{pmatrix} 10 \\ -8 \end{pmatrix}, \begin{pmatrix} -5 \\ 4 \end{pmatrix}, \begin{pmatrix} 15 \\ -12 \end{pmatrix}$

6. $\begin{pmatrix} 3 \\ 6 \end{pmatrix}, \begin{pmatrix} -6 \\ -12 \end{pmatrix}, \begin{pmatrix} 6 \\ 12 \end{pmatrix}$

7. $\begin{pmatrix} 10 \\ 2 \end{pmatrix}, \begin{pmatrix} -5 \\ -1 \end{pmatrix}, \begin{pmatrix} 15 \\ 3 \end{pmatrix}, \begin{pmatrix} -20 \\ -4 \end{pmatrix}$

8. $\begin{pmatrix} -6 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \begin{pmatrix} -10 \\ 0 \end{pmatrix}, \begin{pmatrix} 8 \\ 0 \end{pmatrix}$

9. $\begin{pmatrix} -6 \\ 4 \end{pmatrix}, \begin{pmatrix} 18 \\ -12 \end{pmatrix}, \begin{pmatrix} 3 \\ -2 \end{pmatrix}, \begin{pmatrix} -12 \\ 8 \end{pmatrix}$

10. $\begin{pmatrix} -18 \\ -60 \end{pmatrix}, \begin{pmatrix} 24 \\ 80 \end{pmatrix}, \begin{pmatrix} -3 \\ -10 \end{pmatrix}, \begin{pmatrix} 30 \\ 100 \end{pmatrix}$

EXERCISE 20f (p. 303)

1. $\begin{pmatrix} 7 \\ -1 \end{pmatrix}$

5. $\begin{pmatrix} 10 \\ 0 \end{pmatrix}$

9. $\begin{pmatrix} 7 \\ 8 \end{pmatrix}$

13. $\begin{pmatrix} 7 \\ 10 \end{pmatrix}$

17. $\begin{pmatrix} -2 \\ -4 \end{pmatrix}$

2. $\begin{pmatrix} -8 \\ 2 \end{pmatrix}$

6. $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$

10. $\begin{pmatrix} 6 \\ -4 \end{pmatrix}$

14. $\begin{pmatrix} 10 \\ 0 \end{pmatrix}$

18. $\begin{pmatrix} -5 \\ -2 \end{pmatrix}$

3. $\begin{pmatrix} 7 \\ -4 \end{pmatrix}$

7. $\begin{pmatrix} -6 \\ -6 \end{pmatrix}$

11. $\begin{pmatrix} 6 \\ 9 \end{pmatrix}$

15. $\begin{pmatrix} -1 \\ 11 \end{pmatrix}$

19. $\begin{pmatrix} -8 \\ 5 \end{pmatrix}$

4. $\begin{pmatrix} 2 \\ 6 \end{pmatrix}$

8. $\begin{pmatrix} 3 \\ 6 \end{pmatrix}$

12. $\begin{pmatrix} 7 \\ 11 \end{pmatrix}$

16. $\begin{pmatrix} -2 \\ 10 \end{pmatrix}$

20. $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

EXERCISE 20g (p. 306)

1. a) $\begin{pmatrix} 7 \\ 5 \end{pmatrix}$ b) $\begin{pmatrix} 7 \\ 5 \end{pmatrix}$ c) $\begin{pmatrix} 8 \\ 6 \end{pmatrix}$ d) $\begin{pmatrix} 8 \\ 6 \end{pmatrix}$ e) $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$ f) $\begin{pmatrix} 6 \\ 9 \end{pmatrix}$ g) $\begin{pmatrix} 10 \\ 9 \end{pmatrix}$ h) $\begin{pmatrix} 10 \\ 9 \end{pmatrix}$

2. a) $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ b) $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ c) $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$ d) $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$ e) $\begin{pmatrix} -6 \\ 12 \end{pmatrix}$ f) $\begin{pmatrix} -20 \\ -12 \end{pmatrix}$

3. a) $\begin{pmatrix} 5 \\ 10 \end{pmatrix}$ b) $\begin{pmatrix} 18 \\ 24 \end{pmatrix}$ c) $\begin{pmatrix} 12 \\ 24 \end{pmatrix}$

4. a) $\begin{pmatrix} -19 \\ -1 \end{pmatrix}$ b) $\begin{pmatrix} 4 \\ -11 \end{pmatrix}$

EXERCISE 20h (p. 307)

1. $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$

5. $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

9. $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$

13. $\begin{pmatrix} 5 \\ 10 \end{pmatrix}$

16. $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

2. $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$

6. $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

10. $\begin{pmatrix} -7 \\ 3 \end{pmatrix}$

14. $\begin{pmatrix} 4 \\ -5 \end{pmatrix}$

17. $\begin{pmatrix} -3 \\ 11 \end{pmatrix}$

3. $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$

7. $\begin{pmatrix} 11 \\ 9 \end{pmatrix}$

11. $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$

15. $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$

18. $\begin{pmatrix} -11 \\ 7 \end{pmatrix}$

$$4. \begin{pmatrix} -5 \\ 1 \end{pmatrix} \quad 8. \begin{pmatrix} 5 \\ 8 \end{pmatrix} \quad 12. \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$19. \text{ a) } \begin{pmatrix} 1 \\ 2 \end{pmatrix} \text{ b) } \begin{pmatrix} -1 \\ -2 \end{pmatrix} \quad 20. \text{ a) } \begin{pmatrix} -6 \\ -4 \end{pmatrix} \text{ b) } \begin{pmatrix} -3 \\ -3 \end{pmatrix} \text{ c) } \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

$$21. \text{ a) } \begin{pmatrix} 8 \\ 2 \end{pmatrix} \text{ b) } \begin{pmatrix} 9 \\ 19 \end{pmatrix} \text{ c) } \begin{pmatrix} 0 \\ -22 \end{pmatrix} \text{ d) } \begin{pmatrix} 10 \\ 11 \end{pmatrix} \text{ e) } \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$

$$22. \text{ a) } \begin{pmatrix} -3 \\ 18 \end{pmatrix} \text{ b) } \begin{pmatrix} -3 \\ 0 \end{pmatrix} \text{ c) } \begin{pmatrix} 3 \\ 8 \end{pmatrix} \text{ d) } \begin{pmatrix} 0 \\ -23 \end{pmatrix} \text{ e) } \begin{pmatrix} -4 \\ 22 \end{pmatrix}$$

$$23. \text{ a) } \begin{pmatrix} 5 \\ -10 \end{pmatrix} \text{ b) } \begin{pmatrix} -17 \\ 14 \end{pmatrix} \text{ c) } \begin{pmatrix} 20 \\ -14 \end{pmatrix}$$